A Revenue Enhancing Stackelberg Game for Owners in Opportunistic Spectrum Access

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***Abstract*—A Stackelberg game between three players; spectrum owner, primary users and secondary users is presented under the opportunistic spectrum access (OSA) model, where the secondary users share the channel with primary users in time and secondary user access is performed through a non-perfect listen-before-send scheme. It is shown through simulations that the spectrum owner can enhance her revenue by allowing OSA with a non-zero interference probability to the primary users. The subscription fee of the primary users is lowered in exchange of the non-zero interference probability. The revenue enhancement results from the subscription fees of the secondary users and better utilization of the spectrum. It is also shown through simulations that the enhancement is available for a large range of user preferences such as the value of primary service relative to the secondary, and optimal action of the spectrum owner is robust against estimation errors in these preferences.**

I. Introduction

Wireless spectrum is allocated statically on long term contracts by FCC in the United States and equivalent bodies in other countries, in order to avoid interference from third parties. On the other hand, wireless communication is inherently very dynamic and variable. For example, the user demands change temporally depending on the time of the day or important events going on at a particular period. Wireless channels undergo random fading and shadowing, and mobile devices that operate on batteries might fail. The demand also varies spatially, due to changing population densities. A static allocation has to account for the worst case scenario, which leads to underutilization at other times [1], [2]. Dynamic access of the spectrum can lead to better utilization.

With the advent of smart wireless devices recently, it will soon be possible that secondary networks can coexist with the primary owners of a spectrum with little impact on their performance. A new paradigm called "Opportunistic Spectrum Access (OSA)" has been proposed to this extent [3], [4]. In this paradigm, we refer to the owners of a piece of spectrum as "primary owners (POs)" and to the radios that are on the PO's network as "primary users (PUs)". In addition to these, cognitive radios [5] sense the portions of the spectrum that are not used by the PUs, and utilize these vacancies until a PU shows up, at which point they migrate to another unused spectrum. These types of radios are often referred to as "secondary users (SUs)". The vacancies that are utilized by the SUs can be in time [6], frequency [7], space [8] or combinations of these. In this paper, we consider OSA in time dimension only.

It is well known that perfect sensing of the PU presence is very hard, if not impossible [9], and there is a non-zero probability of interference to the PU radios by the SU radios under the OSA model. Therefore, it has been argued that the POs will never have the incentive to allow OSA, as it will degrade the QoS of PUs (e.g., customers of a PO) and the PO will incur losses in her revenues. On the other hand, we see the spectrum as an economical commodity: whenever a commodity is under-utilized, its owner should be able to increase her revenue by a more efficient utilization. In this paper, we propose a three player Stackelberg game [10] model between the PO, PUs and SUs. We show that by allowing OSA with a non-zero tolerated interference probability to the PUs, the spectrum owner (PO) can enhance her revenue. In exchange for the degraded QoS of the PUs due to the interference from SUs, the PO offers the PUs a lower subscription fee. The enhancement of the revenue comes from the subscription fee of the SUs and the fact that the spectrum is utilized better.

Although a fair amount of recent publications adopted game theory to derive strategic behaviors for cognitive radios under various performance metrics [11]–[14], they don't consider the economical aspect of the OSA model. A few papers bring up revenue maximization and pricing problems of the spectrum owners [15], [16], but they typically don't consider the interference to PUs by SUs in the OSA model. In [17] a very nice revenue enhancing model is proposed where both PU and SU share a set of channels using the same protocol. Blocking a PU call is penalized by a monetary loss, whereas blocking a SU call is not. The authors show that the optimal pricing strategy charges the SUs more when there are less free channels, such that they are less likely to transmit. Our work differs from [17] because we assume the SUs have cognitive radios that sense the idle periods in the channel and utilize these periods, whereas [17] assume the same protocol for both PUs and SUs. Also in our model, the penalty of SUs interfering with PUs is modeled differently. To our best knowledge, this work is the first one that shows economical incentives for the owners under the OSA model where the SU follows a non-perfect listen-before-send access scheme.

We would also like to mention upfront that our model is by no means near perfect. We assume a single channel with

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fixed capacity. A model with multiple channels with variable capacities, where SUs can choose the optimal channels and transmission rates would not only be more realistic, but also it would be more in line with the cognitive radio paradigm. In addition, the SU and PU channel usage models are greatly simplified, as will be explained in the next section. The user utilities are assumed to be equal to their average throughput, which is valid for some applications such as file download but not a good assumption with some other applications such as voice communications. Also, we consider only a single PO (spectrum owner), where multiple competing POs would be a more realistic scenario. We would like to improve our model in the future by taking these concerns into account. However, we believe this work is a reasonable starting point that considers the economical aspect of the OSA model and demonstrates incentives to the spectrum owners to adopt OSA.

The rest of the paper is organized as follows. In Section II, we describe our model and assumptions. Simulations that illustrate enhanced revenues for the owners are presented in Section III.

II. Model and Assumptions

Because we consider OSA in time dimension, we assume that there is one channel with capacity C. The case for multiple channels with variable capacities is in the list of our future work. The owner of this channel is the PO and the regular customers of the PO are PUs. For example in GSM, the channel would be one time slot in one of the 200kHz-wide bands, the PO would be the GSM operator, and the PUs would be the cell phone customers. The channel serves multiple PUs, however, we assume that it is under-utilized, i.e., there exist idle periods where no PU uses the channel. Therefore, the PO allows Opportunistic Spectrum Access (OSA) for enhanced revenues. SUs are allowed to sense the channel and opportunistically utilize it when it is not being used by any PU, while keeping the interference to the PUs below a maximum. This interference constraint will be defined formally shortly.

The PUs pay a fixed subscription fee m_p (in \$ per second) akin to the monthly cell phone fees to the PO. The SUs also pay a fixed subscription fee m_s [1] (in \$ per second). Because the PUs have priority over the SUs for the usage of the channel, they are expected to pay more than SUs for similar throughput. We capture this concept of fairness in subscription fees as follows. We measure the utility of a user with their throughput [2]. We adopt an idealistic continuous view of the channel: if a PU or a SU utilizes the channel x fraction of the time, she achieves a throughput of xC. That is, we ignore the effect of different communication schemes achieving different efficiencies. We also define a parameter α (in bits per \$), which measures the value of \$1 in communicated bits to a typical PU. We assume that for the same amount of channel utilization (therefore the same throughput), a typical PU is willing to pay K times more in subscription fees to the PO, compared to a typical SU. In other words, if we define β as the value of \$1 in communicated bits to a typical SU, $K = \beta/\alpha$. Clearly, $K > 1$.

In the OSA model, the SUs are required to keep the probability of interference to the PUs below a tolerance threshold p_{tol} [6], which is a parameter set by the PO. This non-zero interference probability is required for OSA, as it is shown to be very hard, if not impossible [9], to perfectly detect the presence of the PUs and not interfere with them at all, while achieving non-zero utilization of the channel by SUs.

Given the subscription fee m_p and the interference probability p_{tol}, each PU buys the service from PO with a probability p_p, independently from the other PUs. Similarly, given the subscription fee m_s and interference tolerance constraint p_{tol}, each SU buys the service with probability p_s. Both p_p and p_s are non-increasing functions of m_p and m_s, respectively. However, p_p is a non-increasing and p_s is a non-decreasing function of p_{tol}.

We assume M_p potential PUs exist per channel, thus the average number of PUs served by the channel is equal to $M_p p_p$. The actual number of users is a binomial random variable, however for simplicity, we assume that this number is deterministic and equal to its mean. Similarly, there are M_s potential SUs and the actual number of SUs served by the channel is assumed to be $M_s p_s$. The revenue (per second per channel) of the PO is then given by $R = M_p p_p m_p + M_s p_s m_s$.

What we have described above can be viewed as a three player Stackelberg game [10] (See Figure 1). In this game, the PO is the leader and the PUs and the SUs are the followers. First, the PO sets the values of m_p, m_s and p_{tol}. The PUs and the SUs set p_p and p_s, respectively, in response to PO's actions. The actions of PUs and SUs in turn determine the revenue of the PO. The usage statistics of the PU also effect the action of the SU, which is denoted by the lowermost dashed arrow in Figure 1. As the PO knows the best actions of the PUs and the SUs (p_p and p_s) in response to her actions (m_p, m_s and p_{tol}), she sets them such that her revenues are maximized. The detailed model for each player is explained next.

A. Primary User Model

Each PU generates "calls" independently at a rate q. If the channel is in use by another PU when a call is placed, the new call is blocked. Otherwise, the PU uses the channel for an exponentially distributed period with rate p (i.e., with mean $1/p$). From now on, let us call the channel state to be "busy" when it is in use by a PU and "idle" when it is not. The busy and idle periods of the channel are exponentially distributed random variables with means $B = 1/p$ and $I = 1/(M_p p_p q)$ respectively. With these assumptions, the mean utilization of the channel (fraction of the time that a PU uses the channel) by *one* PU becomes

$$U_p = \frac{q}{p + M_p p_p q}. \tag{1}$$

The SUs sense the channel and opportunistically use it when it is idle, while not violating a maximum probability

[1] Hereafter, subscript p denotes "primary" and subscript s denotes "secondary".

[2] This is not necessarily the most relevant utility metric for some applications, but we chose throughput for simplicity.

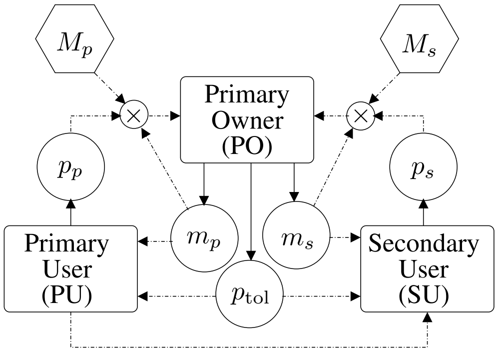

Fig. 1. Three player Stackelberg Game. The PO is the leader and the PU and the SU are the followers. The circled entities with associated solid lines denote the players' actions. The dashed lines denote inputs to the players that affect their actions. M_p and M_s are given and fixed.

of interference p_{tol}. At the worst case, the throughput of the PU is scaled by $1 - p_{\text{tol}}$ due to the SU interference. A more clever SU might back-off, if it senses that the channel is busy after starting transmission. However, for simplicity, we assume the worst case throughput for the PU. With this assumption, the average throughput of a PU is given by

$$\tau_p = C(1 - p_{\text{tol}})\frac{q}{p + M_p p_p q}. \tag{2}$$

The primary users will keep buying the service as long as the average throughput they get meets the subscription fee they pay. For the conversion between throughput and subscription fee, we use the previously defined variable α. At equilibrium, we have

$$C(1 - p_{\text{tol}})\frac{q}{p + M_p p_p q} = \alpha m_p, \tag{3}$$

and the optimal probability at which a PU buys the service is given by

$$p_p = \frac{1}{M_p q}\left(\frac{C(1 - p_{\text{tol}})q}{\alpha m_p} - p\right). \tag{4}$$

Of course, if (4) yields a value less than 0 or greater than 1, then p_p is clipped to 0 or 1, respectively.

B. Secondary User Model

It is important to properly model the impact of SUs on the PU performance. Although both probability and duration of SU interference are important [18], we simply consider the probability of interference, as proposed by Motamedi and Bahai [6]. We use this model for the SU, and for completeness, we briefly summarize this work in this section.

According to [6], the SUs sense the channel and opportunistically use it when it is idle. They satisfy the constraint that the probability of interference to a PU to be lower than p_{tol}. The SU network is assumed to be managed by its own base station, i.e., the whole SU network acts as a single body [3]. The time is divided into slots of length "sensing time" plus the "utilization time" $T_s + T_u$, which are both deterministic. At the beginning of each time slot, the SU network attempts to sense the channel with probability p_a. If attempted, they sense the channel for a period of length T_s. If the channel is decided to be idle, then it is utilized for a period of length T_u. The capacity is shared equally among all SUs, through some kind of multiple access scheme such as TDMA or OFDMA. It is shown in [6] that the probability of interference to the PU is given by

$$p_{\text{int}} = p_a(1-p_{\text{D}})\text{P}(\mathcal{H}_1)+p_a(1-p_{\text{FA}})\text{P}(\mathcal{H}_0)(1-\exp(-T_u/I)), \tag{5}$$

and the average SU utilization U_s (fraction of the time that the SU network uses the channel with no collision with the PUs) is given by

$$U_s = p_a\text{P}(\mathcal{H}_0)(1 - p_{\text{FA}})\exp(-T_u/I)\frac{T_u}{T_u + T_s}, \tag{6}$$

where $\mathcal{H}_0$ and $\mathcal{H}_1$ are the hypotheses that the channel is idle and busy, respectively, p_{D} is the detection probability, and p_{FA} is the false alarm probability during the sensing phase, and $I = 1/(M_p p_p q)$ is defined before as the mean idle time of the channel. The probabilities of the hypotheses are given by

$$\text{P}(\mathcal{H}_0) = \frac{I}{B + I} = \frac{p}{p + M_p p_p q}, \tag{7}$$

$$\text{P}(\mathcal{H}_1) = \frac{B}{B + I} = \frac{M p_p q}{p + M_p p_p q}, \tag{8}$$

where $B = 1/p$ is the mean busy time of the channel as defined before.

We assume T_s, p_{D} and p_{FA} are fixed, however, the SU network optimizes for the variables p_a and T_u to maximize their average utilization U_s while not violating the maximum tolerated interference

$$\begin{aligned} \underset{T_u, p_a}{\text{maximize}} \quad & U_s \\ \text{subject to} \quad & p_{\text{int}} \le p_{\text{tol}} \\ \text{and} \quad & 0 \le p_a \le 1. \end{aligned} \tag{9}$$

[6] performs this optimization numerically. We derived analytical approximations for optimal p_a and T_u. This not only speeds up the simulations of Section III, but also is instrumental to our ongoing work on the analysis of the pareto-optimal frontier of the achievable $\{U_s, p_p\}$ pairs, which is promising to provide insight on the conditions under which the OSA model enhances revenues.

The formulas for the optimal p_a and T_u values fall in 5 different regimes. These regimes and how to compute the optimal p_a and T_u are given as a flowchart in Figure 2 (See

[3] Given that the SU network consists of "cognitive" radios, this management can in practice be performed by a selected cluster head, or by a functionality that is implemented distributedly.

Appendix A for derivation of these formulas). In the figure,

$$T_u^* = \frac{\sqrt{T_s^2 + 4IT_s} - T_s}{2}, \tag{10}$$

$$T_u^\ddagger = \frac{\sqrt{T_s^2 + 4I\gamma T_s} - T_s}{2\gamma}, \tag{11}$$

$$T_u^\dagger = -I \ln\left(\frac{I(\bar{p}_{\rm FA} - p_{\rm tol}) + B(\bar{p}_{\rm D} - p_{\rm tol})}{I\bar{p}_{\rm FA}}\right), \tag{12}$$

$$f(T_u) = \min\left\{1, \frac{p_{\rm tol}(B+I)}{B\bar{p}_{\rm D} + I\bar{p}_{\rm FA}(1 - \exp(-T_u/I))}\right\}, \tag{13}$$

where $\bar{p}_{\rm FA} = 1 - p_{\rm FA}$, $\bar{p}_{\rm D} = 1 - p_{\rm D}$, $\gamma = 1 + \frac{I\bar{p}_{\rm FA}}{B\bar{p}_{\rm D}}$, $B = 1/p$, $I = 1/(M_p p_p q)$ and we used the assumption that $T_u^\ddagger \leq T_u^*$. This assumption is reasonable because usually the detection probability is much bigger than the false alarm probability, and hence $\gamma \gg 1$. For regimes II and V (see Figure 2), we also assumed that $T_u^\ddagger \ll I$ such that $1 - \exp(-T_u^\ddagger/I) \approx T_u^\ddagger/I$. This assumption is also reasonable because in these regimes the maximum tolerated interference $p_{\rm tol}$ is tight, therefore the utilization time usually turns out to be much less than the channel mean idle time. In our simulations, $T_u^\ddagger$ was at most 0.45% of I in these regimes and the maximum error in the approximation was only 0.22%.

Similar to the PU model, the SUs buy the service as long as their average throughput per SU meets its value

$$\frac{CU_s}{M_s p_s} = K\alpha m_s, \tag{14}$$

where K was previously defined as the value of the secondary service relative to primary. Thus

$$p_s = \frac{CU_s}{M_s K\alpha m_s}. \tag{15}$$

Also here, if the right hand side is bigger than 1, p_s is clipped to 1.

At this point, we would like to mention that there is an asymmetry in the PU and SU models in favor of the SU: the SUs can share the channel fairly without any "blocked calls", while the PUs' attempts are blocked if another PU is using the channel. This asymmetry probably skews the results of the simulations in favor of allowing the OSA over not. This assumption however, was made on purpose considering the fact that the SUs consist of cognitive radio devices, which are supposed to exhibit better coordination and traffic awareness whereas the PU network is based on simple devices/protocols.

C. Primary Owner Model

Primary Owner (PO) owns the channel and is the leader of the Stackelberg game. The owner knows that the followers (PUs and SUs) will choose their optimal acceptance probabilities p_p and p_s according to (4) and (15), respectively. Therefore, she adjusts the tolerated interference probability $p_{\rm tol}$ and the subscription fees m_p and m_s to maximize her revenue R

$$\begin{aligned} R &= M_p p_p m_p + M_s p_s m_s \\ &= M_p p_p m_p + \frac{CU_s}{K\alpha}, \end{aligned} \tag{16}$$

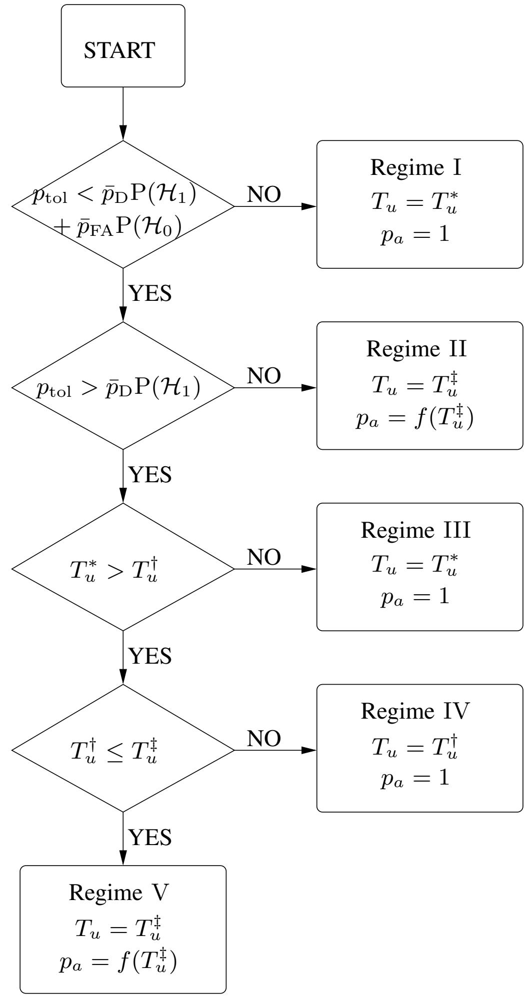

Fig. 2. Flow chart for computing optimal attempt probability and utilization time.

where U_s denotes the average utilization of the channel by the secondary users (Section II-B). Since U_s is not a function of the variables p_s or m_s, without effecting the revenue, we can set $p_s = 1$ and $m_s = \frac{CU_s}{M_s K\alpha}$ using (15) to obtain (16). With this substitution, PO optimization consists of only 2 variables:

$$\begin{aligned} &\underset{p_{\rm tol}, m_p}{\text{maximize}} \;\; R \\ &\text{subject to} \;\; 0 \leq p_{\rm tol} \leq 1 \\ &\text{and} \;\; 0 \leq m_p. \end{aligned} \tag{17}$$

III. SIMULATIONS

In this section, we perform the optimization in (17) numerically, and show that the PO can increase her revenues by allowing OSA under the Stackelberg game model described in Section II [4]. The default values of the parameters we used in the simulations (unless stated otherwise) are given in Table I. When applicable, we used the GSM as a model for

[4]At the time of publication, we didn't know whether the solution found numerically was unique or not.

TABLE I
THE DEFAULT VALUES USED IN THE SIMULATIONS.

Parameter	Explanation	Value
C	Channel capacity	50 Kbps
M_p	Number of potential PUs	50
M_s	Number of potential SUs	50
$B = 1/p$	Mean channel busy time	180s
q	PU call generation rate	10 per day
α	Value of \$1 in bits for a typical PU	5 MB/\$
K	Value of primary service relative to secondary	5
$p_{\rm D}$	PU detection probability (by SU)	0.9
$p_{\rm FA}$	False alarm probability (by SU)	0.01
T_s	Sensing time for PU detection	10 ms

TABLE II
RESULT OF THE PO OPTIMIZATION USING THE PARAMETERS IN TABLE I.

Optimal	w/OSA	w/o OSA
$p_{\rm tol}$	5.31%	–
m_p (monthly)	\$29.84	\$31.5
m_s (monthly)	\$5.89	–
PO revenue (monthly, per channel)	\$1,786.4	\$1,575.4
SU utilization	47.6%	–
PU acceptance prob. (p_p)	1	1

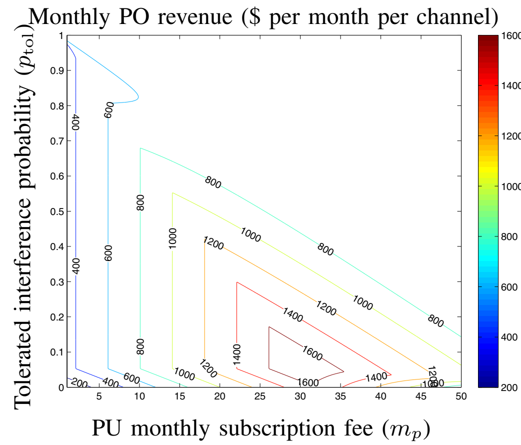

Fig. 3. Contours of the PO revenue using parameters in Table I.

choosing these numbers. For example, 50 Kbps is about the maximum data rate of the EDGE standard, which is used in GSM. Similarly, 3 minutes (180 seconds) is a reasonable mean phone call time. For choosing the number of potential PUs and SUs, we considered the fact that about 750,000 people live in the San Francisco County [19], which has about 20 GSM towers [20]. There are about 1000 channels in GSM, which results in 37.5 people per channel per tower. We simply rounded this to $M_p = M_s = 50$. The values for the rest of the parameters are chosen arbitrarily. We assume that, given the sensing hardware of the cognitive radios of SUs, $p_{\rm D}$, $p_{\rm FA}$ and T_s are fixed. In reality, the detection and false alarm probabilities are a function of the sensing time [6], [9], however, we ignore this fact.

In Figure 3, we plot the contours of the PO revenue with respect to $p_{\rm tol}$ and m_p obtained with parameters in Table I. As you can see, the maximum revenue is achieved for a non-zero tolerated interference probability. We report the optimal values for the actions of the PO and the resulting revenues for the cases with and without OSA in Table II. By allowing 5.31% probability of interference and lowering the PU subscription fee by about \$2 per month, the PO can increase her revenue by 13.4% by charging the secondaries a monthly fee of \$5.89. Moreover, the optimal acceptance probability of a PU is 1 for both cases, which means the PO doesn't lose her primary customers. When OSA is allowed, the channel is used by PUs 51% of the time, and the utilization by the SU network is 47.6%. This demonstrates that the channel is nearly fully utilized at optimum.

In Section II, we defined K as the value of the primary service relative to the secondary service for typical users. The knowledge of this parameter is key to the spectrum owner in the decision of allowing OSA. This parameter can only be found by customer surveys, which might result in some confidence interval rather than an exact value. The spectrum owner will then have the natural questions:

- For what range of K values is OSA more profitable?
- What is the gain in revenue for different values of K?

Thus next, we analyze how key parameters change with respect to K. We also do this for 3 different PU call generation rates (q), namely 5, 10 and 20 calls per day.

Note that knowledge of the parameter α is also required for the decision of OSA and it can also be found by surveys. However for brevity, we chose to run our simulations by varying K only. Note that K was defined as the ratio of 2 parameters, namely β, the value of 1\$ in communicated bits for a typical SU, over α, the value of 1\$ in communicated bits for a typical PU. Therefore, varying K in simulations captures the affect of varying α for a fixed β.

Figure 4 plots the gain in the revenue versus K. The higher K is, the less SUs are willing to pay. When the channel is utilized highly by the PUs (20 calls per day), the PO can increase revenues by allowing OSA for $K < 8$. For medium PU utilization (10 calls per day), the revenues increase for $K < 20$ and for sparse PU utilization (5 calls per day), the revenues increase for $K < 50$. This makes sense, because according to the model we use, once the SUs buy the service, they utilize the channel maximally without exceeding the interference tolerance. Therefore, the less the channel is utilized by the PUs, the more the SUs will utilize it and the larger the increase in revenue will be, compared to no OSA. But of course, the actual revenues (not shown) are larger for higher PU utilization, as the secondary service costs less ($K > 1$).

In Figure 5, we plot the optimal tolerated interference probability ($p_{\rm tol}$) vs. K. In this figure, the regions for K where OSA increases the revenues for PO (i.e., $p_{\rm tol} > 0$) is also evident, in agreement with the previous plot. It is noticed that the optimal $p_{\rm tol}$ is larger (when positive) for larger PU traffic (larger q). This can be explained as follows. We observed that, when OSA increases the revenue, PO wants to set the variables such that the optimal utilization time (T_u) and attempt probability (p_a) of the SUs fall in region

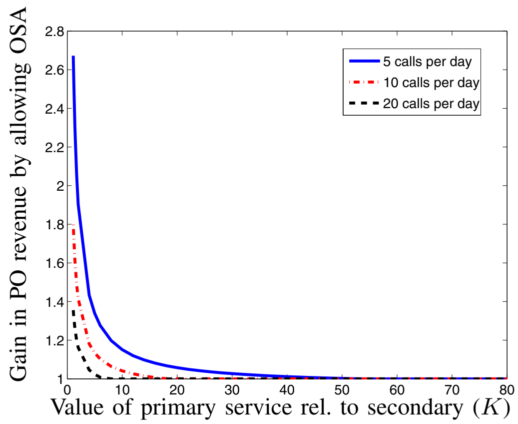

Fig. 4. Gain in the PO revenue vs. K for 3 different levels of PU channel usage statistics. Other parameters are given in Table I.

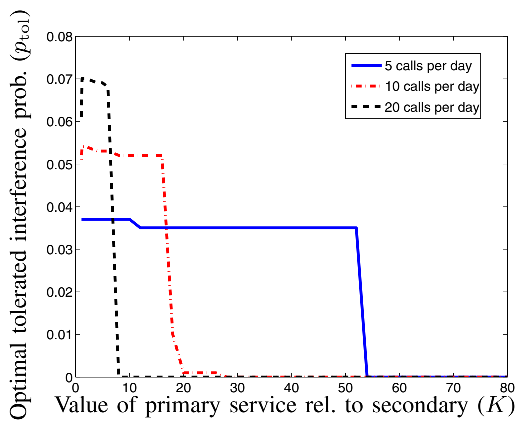

Fig. 5. Optimal tolerated interference level p_{tol} vs. K for 3 different levels of PU channel usage statistics. Other parameters are given in Table I.

IV (see Figure 2). For this to happen, p_{tol} has to exceed $(1-p_{\mathrm{D}})\mathrm{P}(\mathcal{H}_1)$, which is equal to 3.42%, 5.10% and 6.76% for $q = 5$, 10 and 20 calls, respectively. In Figure 5, the optimal p_{tol} are just above these values, when OSA is allowed. This is also intuitive, as the PUs utilize the channel more, the PO has to increase the interference tolerance, otherwise the SUs cannot utilize the channel.

Also in the figure for $q = 10$ and $K > 20$, there is a small region with non-zero p_{tol}. In this region, p_{tol} is very small and the revenue increase is insignificant. Therefore, we practically assume that this belongs to the case where OSA doesn't increase the revenue.

Next, we plot the optimal PU subscription fee (m_p) vs. K (Figure 6). When K is very close to 1, it means SUs value the service as highly as the PUs. Therefore the PO can earn more from the SUs, because they can utilize the channel better (they share the channel fairly, while PUs get blocked if another PU is using the channel, as explained before). Therefore, it is better to allow OSA and keep the PU subscription fee high. As soon as K goes away from 1 (which is arguably a more reasonable region for K), m_p drops below the level when OSA is not allowed and $p_{\mathrm{tol}} = 0$. As OSA is not anymore profitable and p_{tol} drops to 0, optimal m_p goes up.

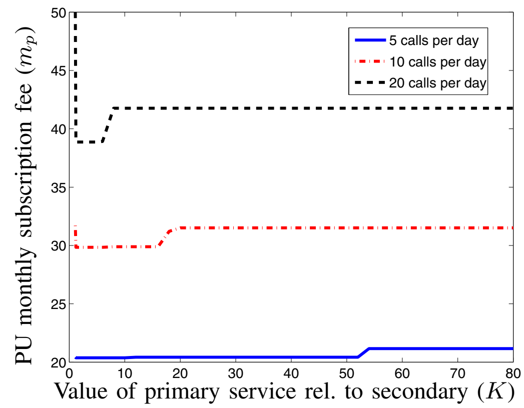

Fig. 6. Optimal PU subscription fee vs. K for 3 different levels of PU channel usage statistics. Other parameters are given in Table I.

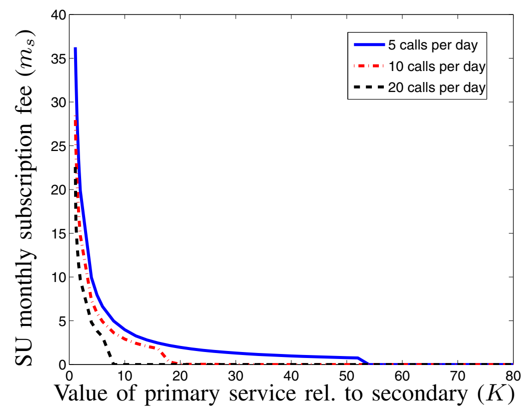

Fig. 7. Optimal SU subscription fee vs. K for 3 different levels of PU channel usage statistics. Other parameters are given in Table I.

Next, we plot optimal SU subscription fee (m_s) vs. K (Figure 7). As expected, it decreases with increasing K. It also increases with decreasing PU utilization; as SUs can utilize the channel better, hence achieve better throughput. This is also visible in the following graph, where the resulting SU utilization is plotted vs. K (Figure 8).

One important observation from Figures 5 and 6 is that p_{tol} and m_p stay fairly constant with respect to K when OSA is allowed, which provides robustness against errors in the estimation of the parameter K. In Figure 7, m_s does not show this characteristic, however as explained in Section II-B, m_s is selected such that $p_s = 1$. If the PO overshoots the m_s, p_s will go down without effecting $m_s p_s$ product, and hence the revenue. This is not true if the PO undershoots the optimal m_s, however. Nevertheless, the choice of optimal PO action (p_{tol}, m_p and m_s triplet) is clearly robust against errors in the estimation of the value of the primary service relative to the secondary (K).

IV. Conclusion

We analyzed the opportunistic spectrum access (OSA) model, where the secondaries share the channel with primary

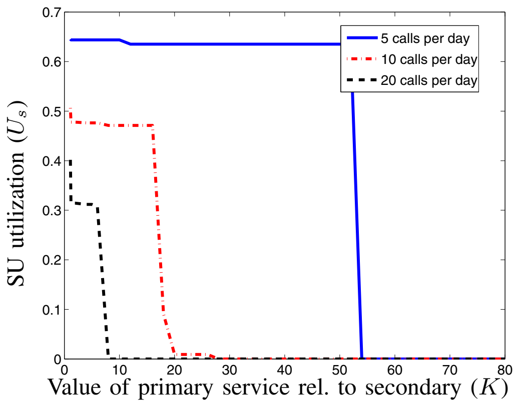

Fig. 8. SU utilization achieved with optimal p_{tol}, m_p and m_s of the previous figures.

users in time and secondary user access is performed through a non-perfect listen-before-send scheme. We presented incentives through revenue enhancement for the spectrum owners to adopt this model. We proposed a three player Stackelberg game between the spectrum owner, the primary users and the secondary users. We showed in simulations that, by allowing OSA with a non-zero tolerated interference probability to the primary users, the spectrum owner can enhance her revenue. In exchange for the degraded QoS of the PUs due to the interference from SUs, the PO offers the PUs a lower subscription fee. The enhancement of the revenue comes from the subscription fee of the SUs and the fact that the spectrum is utilized better. We demonstrated that various levels of enhancements in the revenue are available to the spectrum owners for a large range of user preferences such as the value of primary service relative to the secondary, and the optimal choice of actions for the spectrum owner are robust against estimation errors in these preferences.

Although we believe that our model is a reasonable starting point that considers the economical aspect of the non-perfect listen-before-send OSA model and demonstrates incentives to the spectrum owners to adopt OSA, it is still overly simplified. In the future, we are planning to improve our model by considering multiple channels with variable capacities and more realistic primary and secondary channel usage statistics. Also, we are planning to compare different utility metrics suitable for a wide variety of applications, rather than just assuming the average throughput. In addition, we are working on the analysis of the pareto-optimal frontier of the achievable secondary user utilization and primary user acceptance probability doublets. This work is going to provide insights on the conditions when OSA is more profitable than not allowing OSA. Finally, we are going to examine the effect of competition among multiple spectrum owners and collaboration within the secondary network to the secondary utilization and owner revenue under the OSA model.

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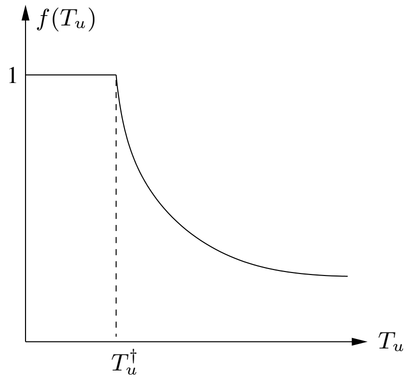

Fig. 9. The function f vs. utilization time T_u in (18).

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APPENDIX A
DERIVATION OF OPTIMAL UTILIZATION TIMES AND ATTEMPT PROBABILITIES

Note that in (6), U_s is an increasing function of p_a. Therefore, optimal p_a in (9) is given by [6]

$$p_a = f(T_u) = \min\left\{1, \frac{p_{\text{tol}}(B+I)}{B\bar{p}_{\text{D}} + I\bar{p}_{\text{FA}}(1-\exp(-T_u/I))}\right\}. \quad (18)$$

We can substitute this formula into (6) and perform the optimization with respect to T_u only. Once optimal T_u is found, optimal $p_a = f(T_u)$.

The function "$f(T_u)$" is depicted in Figure 9. As illustrated in the figure, let us define $T_u^\dagger$ as:

$$T_u^\dagger \triangleq \sup\{T_u;\ f(T_u) = 1\}. \quad (19)$$

The value of $T_u^\dagger$ is given by

$$T_u^\dagger = -I\ln\left(\frac{I(\bar{p}_{\text{FA}} - p_{\text{tol}}) + B(\bar{p}_{\text{D}} - p_{\text{tol}})}{I\bar{p}_{\text{FA}}}\right). \quad (20)$$

Assuming $T_u^\dagger > 0$, we define three sets that the optimal utilization time can belong to:

Case 1: $T_u \in [0, T_u^\dagger)$

Case 2: $T_u \in \{T_u^\dagger\}$

Case 3: $T_u \in (T_u^\dagger, \infty)$.

If Case 1 is valid, we have $p_a = 1$ (since $f(T_u) = 1$) and

$$U_s = \bar{p}_{\text{FA}}\exp(-T_u/I)\frac{T_u}{T_u+T_s}\frac{I}{B+I}. \quad (21)$$

By differentiating this with respect to T_u and equating to zero, we obtain the optimal utilization time for this case. Let us call it T_u^*:

$$T_u^* = \frac{\sqrt{T_s^2 + 4IT_s} - T_s}{2}. \quad (22)$$

If Case 3 is valid, $p_a = \frac{p_{\text{tol}}(B+I)}{B\bar{p}_{\text{D}} + I\bar{p}_{\text{FA}}(1-\exp(-T_u/I))}$ and

$$U_s = p_{\text{tol}}\frac{T_u}{T_u+T_s}\frac{\bar{p}_{\text{FA}} I\exp(-T_u/I)}{B\bar{p}_{\text{D}} + I\bar{p}_{\text{FA}}(1-\exp(-T_u/I))}. \quad (23)$$

Differentiating with respect to T_u yields

$$\frac{\partial U_s}{\partial T_u} = p_{\text{tol}}\frac{T_s}{(T_u+T_s)^2}\frac{\bar{p}_{\text{FA}} I\exp(-T_u/I)}{B\bar{p}_{\text{D}} + I\bar{p}_{\text{FA}}(1-\exp(-T_u/I))}$$
$$-p_{\text{tol}}\frac{T_u}{T_u+T_s}\frac{\bar{p}_{\text{FA}}\exp(-T_u/I)(B\bar{p}_{\text{D}} + I\bar{p}_{\text{FA}})}{(B\bar{p}_{\text{D}} + I\bar{p}_{\text{FA}}(1-\exp(-T_u/I)))^2} = 0. \quad (24)$$

The solution of this equation is the the optimal T_u for this case. In order to be able to get a closed form solution, we use the assumption that $T_u \ll I$ such that $1-\exp(-T_u/I) \approx T_u/I$ as explained in Section II-B. Let us call the optimal usage time for this case $T_u^\ddagger$:

$$T_u^\ddagger = \frac{\sqrt{T_s^2 + 4I\gamma T_s} - T_s}{2\gamma}, \quad (25)$$

where $\gamma = 1 + \frac{I\bar{p}_{\text{FA}}}{B\bar{p}_{\text{D}}}$.

With the assumption of $T_u^\ddagger \le T_u^*$ (see Section II-B), there are 3 orderings of $T_u^\dagger$, T_u^* and $T_u^\ddagger$. Each one of these orderings corresponds to one of the cases mentioned above, which in turn corresponds to one of the regimes in Figure 2. For example, if the ordering $T_u^\ddagger \le T_u^* \le T_u^\dagger$ is valid, then T_u^* is the unique optimizer; since U_s is continuous for $T_u \in \mathbb{R}_+$, differentiable for $T_u \in \mathbb{R}_+ \setminus \{T_u^\dagger\}$, decreasing for $T_u \in (T_u^\dagger, \infty)$, and T_u^* is the unique solution of $\frac{\partial U_s}{\partial T_u} = 0$ in $[0, T_u^\dagger)$. This corresponds to Case 1 and Regime III. If the ordering $T_u^\ddagger \le T_u^\dagger \le T_u^*$ is valid, U_s is increasing for $T_u \in (0, T_u^\dagger)$ and decreasing for $T_u \in (T_u^\dagger, \infty)$ and hence, $T_u^\dagger$ is the unique optimizer. (Case 2, Regime IV). If the ordering $T_u^\dagger \le T_u^\ddagger \le T_u^*$ is valid, then $T_u^\ddagger$ is the unique optimizer with similar reasoning (Case 3, Regime V).

Regimes I and II in Figure 2 correspond to the case when $T_u^\dagger > 0$ assumption does not hold, which means the argument of the natural logarithm in (20) is out of (0,1). When the argument is greater than or equal to 1, we have $p_{\text{tol}} \le \bar{p}_{\text{D}}\text{P}(\mathcal{H}_1)$, which keeps the second argument of the min operator in (18) always less than or equal to 1. Therefore, optimal T_u is given by $T_u^\ddagger$. This is called Regime II in Figure 2. When the argument of the logarithm is non-positive, it means that the tolerated interference probability is so high that, it cannot be violated ($p_{\text{tol}} \ge \bar{p}_{\text{D}}\text{P}(\mathcal{H}_1) + \bar{p}_{\text{FA}}\text{P}(\mathcal{H}_0)$). Therefore, optimal p_a is 1 and optimal T_u is given by T_u^*. This is called Regime I in Figure 2.

In addition to the above, we need the assumptions that $B > 0$, $I > 0$, $p_{\text{tol}} > 0$, $T_s > 0$, $p_{\text{FA}} < 1$, in the uniqueness arguments. The boundary cases that are observed in simulations are as follows: when $p_p \to 0$, no PU buys the service. Therefore, $I \to \infty$ and optimal $T_u \to \infty$ and $p_a \to 1$. When $p_{\text{tol}} = 0$, OSA is not allowed and optimal $T_u = 0$, $p_a = 0$.